

change is small and the shock can be treated as a simple wave to which either eq. (35) or (36) applies. In this approximation the interactions of shock waves and rarefactions can be calculated from eqs. (25) and (26).

### 3. - Elementary wave interactions.

Equations (32), (34), (35) and (36) uniquely define and limit the values of particle velocity,  $u$ , which can be achieved by simple shock or rarefaction from a given state  $(p_0, V_0, u_0)$ . This limitation on states which can be reached in a single wave transition supplies a powerful tool for thinking about and calculating the fields of high-amplitude waves. The problem is transformed into a «hodograph» plane in which the variables are  $(u, p)$ ,  $(u, l)$ ,  $(r, s)$  or some equivalent set. We shall use  $u, p$  here because of continuity conditions on  $u$  and  $p$  at an interface or boundary. The significance of this choice will appear later.

Various useful representations of a shock and of a rarefaction are shown in Fig. 4. In 4 *a*) is a cross-section of a half-space to which a pressure  $p_1$  was applied at  $t=0$  and released at  $t=t_0$ . The pressure profile at this particular  $t > t_0$  is shown in 4 *b*). It consists of a forward-facing shock, designated  $\mathcal{S}_+$ , a region of uniform pressure  $p_1$  and particle velocity  $u_1$ , and a rarefaction  $\mathcal{R}_+$ . The notations  $\mathcal{S}$  and  $\mathcal{R}$  are introduced here to denote shock and rarefaction waves, respectively. Forward-facing waves are denoted by the subscript «+», backward-facing by «-». In Fig. 4 *c*) the flow is shown in the  $(x, t)$  plane. Region I is the uniform initial state  $(p_0, V_0, u_0)$  with  $u_0 > 0$ . The shock front,  $\mathcal{S}_+$ , has constant slope until the following rarefaction overtakes it, reducing its amplitude and velocity. Region II is the uniform state  $(p_1, u_1, V_1)$  behind the shock. Region III is the rarefaction  $\mathcal{R}_+$  in which pressure and particle velocity are diminishing. Region IV is again at the ambient pressure  $p_0$  but volume and particle velocity now differ from  $V_0$  and  $u_0$ . The path  $OAB$  is the trace of the half-space surface, sometimes called the «piston path»,  $\mathcal{P}$ . The dashed curve is the path of a single particle or mass element traversed successively by  $\mathcal{S}_+$  and  $\mathcal{R}_+$ . Figure 4 *d*) shows the wave process in the  $(p, V)$  plane. The initial shock compression is along the Rayleigh line to the state  $B$  on the Hugoniot. The rarefaction, assumed to be isentropic, expands the material along the dashed isentrope to the final state  $C(V'_0, p_0, u'_0)$ . In Fig. 4 *e*) the process is shown in the  $(p, u)$  plane. The straight line  $AB$  with slope  $dp/du = \rho_0(D - u_0)$  is the image of the Rayleigh line. The compressed state  $B$  lies on the image of the Hugoniot curve and the dashed curve  $BC$  is the image of the isentrope of Fig. 4 *d*). Because the shock process is entropic and because most materials have positive thermal expansion coefficients, the final state  $(u'_0, p_0)$  is normally to the left of  $(u_0, p_0)$  for forward-facing waves.

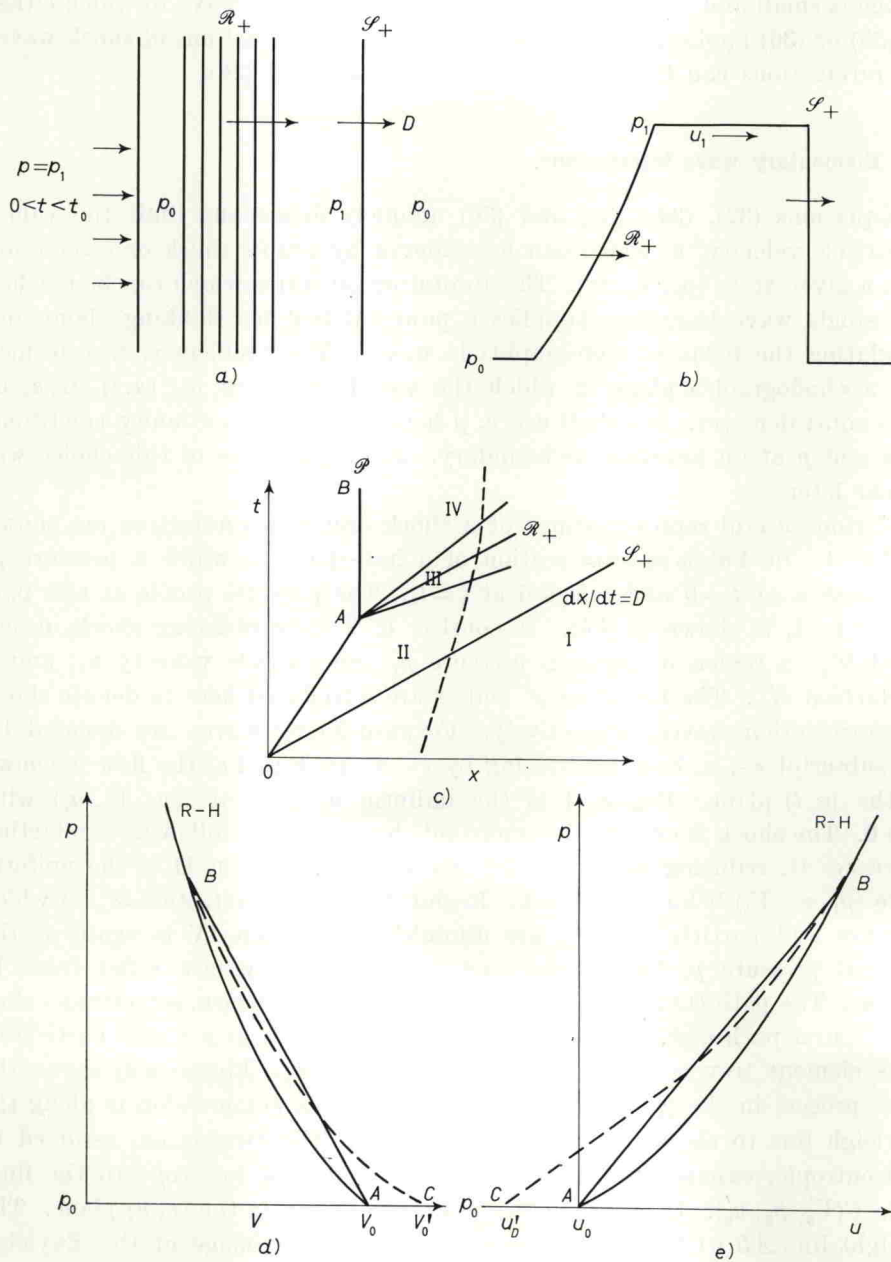


Fig. 4. - Forward-facing rarefaction overtaking a shock. a) planes of constant phase in half-space; b) pressure profile,  $t > t_0$ ; c)  $(x-t)$  diagram; d)  $(p-V)$  diagram; e)  $(p-u)$ -plane.